

Midterm 2 – 70 Points

You must answer all questions. Please write your name on every page. The exam is closed book and closed notes. You may use calculators, but they must not be graphing calculators. Do not use your own scratch paper.

You must show your work to receive full credit

I have neither given nor received unauthorized aid on this examination, nor have I concealed any similar misconduct by others.

Signature _____

1. (45 Points) Suppose that you estimate sales by a firm as a function of the average education level of employees.

$$\log(\text{Sales}) = \beta_0 + \beta_{\text{educ}} \text{education} + u$$

Sales is measured in dollars per year, and *education* is measured in average years of post-secondary education across employees within the firm.

- a.) Suppose you estimate $\hat{\beta}_{\text{educ}} = 0.07$. Please interpret this estimate. (5 Points)

On average, a one year increase in average education at the firm yields a 7% increase in sales

- b.) Supposing again that $\hat{\beta}_{\text{educ}} = 0.07$, and that the covariance between *education* and $\log(\text{Sales})$ is 2, what is the variance of *education* within the sample? (5 points)

$$\hat{\beta}_{\text{educ}} = 0.07 = \frac{2}{\sigma_{\text{educ}}^2} \quad \sigma_{\text{educ}}^2 = \frac{2}{0.07} = 28.6$$

-2 if they solved for SD

c.) Please state the four assumptions, and only four, that are required for unbiased estimates. (5 Points)

1. Linear in parameters +1
2. Random sample +1
3. $E(u|X) = 0$ (or zero conditional mean) +1
4. $\sigma_u^2 > 0$ +1

+1 if they did not write homoskedasticity

d.) In running the regression, I forgot to include *technology*, which is a variable measuring the technology level of the firm's products. If *technology* is associated with higher sales, and higher technology products require a more educated workforce (i.e. they are positively correlated), in what direction, if any, is the estimate $\hat{\beta}_{educ}$ biased? Supposing that the original estimate of $\hat{\beta}_{educ}$ is positive, what can be said about the sign of β_{educ} ? (5 Points)

$$\log(\text{Sales}) = \underbrace{\text{educ} + \text{tech}}_{+} + u \quad +3$$

Bias is positive/upward

We cannot say anything about the ~~bias~~^{sign} of β_{educ} , since $\beta_{educ} < E(\hat{\beta}|X)$ and $\hat{\beta} > 0$. +2

e.) Suppose that I forgot to include the variable, *Holiday*, which is a dummy variable marking the Holiday period during which sales are usually larger. Suppose I say that the bias from forgetting about this variable is negative. What does this imply about the correlation between *education* and *Holiday*? Do you think this implies that we hire higher or lower quality workers during the holiday period? (5 Points)

$$\log(\text{Sales}) = \underbrace{\text{educ} + \text{holiday}}_{+} + u \quad +3$$

Bias negative $\Rightarrow \text{corr}(\text{educ}, \text{holiday}) < 0$

We hire lower quality workers during holidays.

+2

f.) Please describe what it means for errors to be homoskedastic. (5 points)

The variance of the error term is unrelated to x +5
 +3 if they just wrote the equation

g.) I report that the R^2 for the above regression is 0.10. What does this say about the model? What does this not say about the model? (5 points)

- The model matches 10% of the variation in the dependent variable +3
 - This says nothing about causality. +2

h.) Suppose that I double the size of the sample by replicating all observations once. What happens to the estimated variance of $\hat{\beta}_{educ}$ and why? (5 points)

$$\hat{\sigma}_\beta^2 = \frac{\frac{1}{n-2} \sum_{i=1}^n \hat{u}_i^2}{\sum_{i=1}^n (x_i - \hat{\mu}_x)^2} = \frac{\frac{1}{2n-2} \sum_{i=1}^{2n} \hat{u}_i^2}{2 \sum_{i=1}^n (x_i - \hat{\mu}_x)^2} = \frac{n-2}{2n-2} \frac{1}{n-2} \frac{\sum_{i=1}^n \hat{u}_i^2}{\sum_{i=1}^n (x_i - \hat{\mu}_x)^2} = \frac{n-2}{2n-2} \hat{\sigma}_\beta^2 < 1$$

sample size increases +3
 Variance of $\hat{\beta}_{educ}$ falls +2

i.) What is the most important thing you will learn in this class? (5 Points)

Causality +5
 all or nothing

2. (25 Points) Using a sample of cars sold in California, I wish to estimate

$$MPG = \beta_0 + \beta_{GVW}GVW + u$$

MPG is miles per gallon of gas and GVW is gross vehicle weight measured in thousands of pounds.

a.) Suppose you estimate that $\hat{\beta}_0 = 70$. Please interpret this estimate. Is this a sensible estimate to be interested in? (5 Points)

+3 A vehicle with 0 weight has a predicted MPG of 70.

+2 No, not sensible. There exists no such variable.

b.) Suppose that $\hat{\beta}_{GVW} = -10$ using our first sample, but the true population value β_{GVW} is -15. Is this a problem? If our four assumptions for unbiasedness are satisfied, what must be the case for future estimates of $\hat{\beta}_{GVW}$ if we continually resample and estimate this relationship? (5 Points)

Not a problem. +1

Future estimates must be centered around -15
or
At least one future estimate must be below -15 +4

c.) There are 0.45 kilograms per pound (approximately). If I measure GVW in kilograms, what does this do to the estimate $\hat{\beta}_{GVW} = -10$? (5 Points)

$$\begin{aligned} MPG &= \beta_0 + \beta_{GVW} \text{ GVW lbs} + u \\ &= \beta_0 + \beta_{GVW} \text{ GVW kg} \cdot \left(\frac{1 \text{ lb}}{0.45 \text{ kg}}\right) + u \end{aligned}$$

some work or explanation
+3

Estimate is changed to $\hat{\beta}_{GVW} \cdot \frac{1}{0.45} = \underline{22.2}$ +2

d.) Suppose instead that I estimate

$$\log(\text{MPG}) = \beta_0 + \beta_{\text{GVW}} \log(\text{GVW}) + u$$

Please **derive** using derivatives the interpretation for $\hat{\beta}_{\text{GVW}}$. Please state in words how we interpret $\hat{\beta}_{\text{GVW}}$.
(10 Points)

$$\frac{d \text{MPG}}{\text{MPG}} = \beta_{\text{GVW}} \frac{d \text{GVW}}{\text{GVW}}$$

$$\Rightarrow \beta_{\text{GVW}} = \frac{\frac{d \text{MPG}}{\text{MPG}}}{\frac{d \text{GVW}}{\text{GVW}}}$$

+5

β_{GVW} is the elasticity of MPG with respect to GVW

OR

A 1% change in GVW yields a β_{GVW} % change in MPG.

+5

only 3 point if it
looks like the wrong answer
was simply wrong.

Please enjoy Halloween safely.